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Relational Contracts and Job Design*

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Abstract

This paper analyzes the problem of optimal job design when there is only one contractible and imperfect performance measure for all tasks whose contribution to firm value is non-verifiable. I find that task splitting is optimal when relational contracts based on firm value are not feasible. By contrast, if an agent who performs a given set of tasks receives an implicit bonus, the principal always benefits from assigning an additional task to this agent.

Keywords: job design, multi-tasking, relational contracts

JEL classification: M51, M54

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1 Introduction

Measuring employee performance is often difficult because objective performance measures only imperfectly reflect an employee's true contribution to the firm. Thus, if rewards depend on imperfect measures, employees' incentives are not perfectly aligned with the firm's objectives.¹ The use of subjective performance measures, i.e., measures that are observed only by the contracting parties, may mitigate this problem. Indeed, subjective performance evaluation plays an important role in incentive contracting (see, e.g., Gibbons (2005)). Lincoln Electric, for example, motivates its workforce by using piece rates in combination with bonuses based on supervisors' subjective assessments. Thereby, workers are not only rewarded for high output but also for more complex and subtle achievements such as cooperation, innovation, or dependability. Furthermore, Hayes and Schaefer (2000) find empirical evidence that there is subjective assessment in the determination of salary and bonus of chief executive officers.

Informal agreements based on subjective performance evaluation cannot be part of an enforceable (or explicit) employment contract but have to be self-enforcing. This may be the case if the principal cares about its reputation in future relationships (Holmström (1981), Bull (1987)). Baker et al. (1994) show that explicit and relational contracts² can be complements as well as substitutes. While in some circumstances only a combination of explicit and relational contracts generates non-negative profits, relational contracts are infeasible if objective performance measures are sufficiently close to perfect.³

¹See, e.g., Kerr (1975) for an extensive number of examples.

²The term relational contract denotes an informal agreement that is not enforceable by a court but is self-enforcing. Such contracts are also called "implicit" (Baker et al. (1994), MacLeod and Malcomson (1989)), "self-enforcing" (Klein (1996)) or "self-enforcing implicit" (Bull (1987)).

³For the interaction of explicit and relational contracts see also Pearce and Stacchetti (1998), Bernheim and Whinston (1998), Che and Yoo (2001), and Demougin and Fabel (2004). More generally, contributions to the theory of relational contracts include e.g., MacLeod and Malcomson

The aim of this paper is to investigate how the possibility to engage in relational contracts affects optimal job design, i.e., the optimal grouping of tasks into jobs. To do so, I first reformulate the model by Baker et al. (1994) for the case of multiple tasks. I assume that all tasks jointly affect non-verifiable firm value and a contractible but imperfect performance measure.

For a given set of tasks performed by a single agent, I find that relational contracts are feasible if the performance measure is sufficiently distorted⁴ or firm value is sufficiently responsive to changes in effort. In both cases, the principal greatly benefits from better aligning incentives by paying an implicit bonus based on firm value. Employees anticipate that it is in the principal's interest not to renege on relational contracts since she wants to retain the possibility of using them in future periods.

In the next step, I examine when tasks should be split between agents. In my model, the only externality that can arise is due to the misallocation of effort across tasks. As a result, the first-best solution is implemented if the principal employs one agent for each task because this prevents misallocation of effort. Thus, in my framework, the solution to the job design problem is nontrivial only if employing one agent for each task is not possible. This is for example the case if some tasks are non-separable, e.g., quantity and quality in the production of a good.⁵ For simplicity, I consider an environment in which three tasks are to be assigned to either one or two agents. Furthermore, agents cannot simultaneously perform the same task. Therefore, task splitting denotes the grouping of tasks in two different jobs where no task is part of both jobs.

Task splitting has two effects: On the one hand, effort in the one-task job is

(1989), Klein (1996), Baker, Gibbons, and Murphy (2002), MacLeod (2003), and Levin (2003).

⁴This result is in line with Baker et al. (1994).

⁵Another possibility is that agents have positive opportunity costs. Since the principal must compensate agents for their opportunity costs, she might prefer to employ fewer agents than there are tasks.

first-best. On the other hand, the agent in the two-task job receives a lower implicit bonus than an agent performing all three tasks would receive. By withdrawing a task from an agent, this agents' performance becomes less important for the firm value. Therefore, the principals' temptation to renege on a relational contract increases, leading to a lower feasible implicit bonus. This result always holds, even though an agent performing only two tasks may have more distorted explicit incentives than an agent performing three tasks.

The principal prefers to split tasks if she cannot commit herself to paying an implicit bonus to an agent performing all three tasks. Although implicit bonuses remain infeasible under task splitting, the effect of setting first-best incentives for the single-task job increases the principal's expected profit. By contrast, if the principal can commit to paying an implicit bonus to an agent who performs two tasks, expected profit increases if the third task is also assigned to this agent. This leads to a strengthened relational contract, which outweighs the loss from not having first-best incentives for the third task.

Thus, the results suggest that task assignments should be more complex when well aligned objective performance measures are not available or firm value is highly responsive to changes in effort, because then relational contracts are feasible. Frequently, this is the case on higher hierarchy levels within the firm. Then, jobs should be designed such that production workers specialize in a narrow range of tasks, while managers perform a broader range. The former group is more likely to be paid according to pure explicit contracts, while the latter one tends to be rewarded by a combination of explicit and relational contracts.

Furthermore, I also analyze how tasks should be grouped into jobs if all three tasks are separable and have to be split between two agents. For example, due to lack of time, it might not be possible that one agent performs all tasks. Since effort in the one-task job is first-best, one would expect that it is efficient to assign the task that affects firm value most strongly to this job. However, whether this is true or

not depends on the characteristics of the corresponding two-task job. For example, if the agent who performs two tasks receives an implicit bonus, the principal may even want to assign the two most important tasks to him to be able to commit to a high-powered relational contract.

Agents are risk-neutral and have unlimited liability. However, it can be shown that a limited liability constraint for agents affects expected profits under task splitting and no task splitting in the same way. Therefore, the results regarding the principal's optimal decision on task splitting also hold if agents are protected by limited liability.

I further assume that agents' opportunity costs of working for the firm are zero so that there are, a priori, no additional costs of employing more than one agent. The extension to positive opportunity costs is discussed in section 6. Finally, most of the results can be generalized to the case of splitting n tasks between less than n agents, which I also explain in section 6.

With respect to job design, the problem considered in this paper is most closely related to Itoh (1994, 2001). He also shows that it is often optimal to group a broad range of tasks into an agent's job. In his framework, there is also one joint performance measure for all tasks. However, agents are risk-averse and the degree of cost substitutability between tasks varies. Assigning all tasks to one agent is optimal when the degree of substitutability is sufficiently low because then the effect of paying only one risk premium dominates.⁶ By contrast, I focus on the impact of relational contracts in an environment with risk-neutral agents and independent tasks.

Among the first contributions to multi-tasking and job design are Holmström and Milgrom (1991) and Itoh (1991, 1992).⁷ They study static settings with risk-averse

⁶Moreover, Itoh (1994, 2001) also investigates under which circumstances the principal prefers to perform a task by herself.

⁷In particular, Itoh (1991a, 1992) examines when it is optimal to induce cooperation in multi-agent situations.

agents and one performance measure for each single task. Meyer, Olsen, and Torsvik (1996) and Olsen and Torsvik (2000) extend the model by Holmström and Milgrom (1991) to a dynamic setting with limited intertemporal commitment of the principal. While focussing on the "ratchet effect", they show that rules for optimal job design in a static setting (such as sole responsibility for tasks, grouping hardest-to-monitor tasks in one job and easiest-to-monitor tasks in another, or a positive correlation between discretion and incentives) may no longer hold in a multi-period framework. This is also the case in my model in which task splitting is always optimal in a one-shot relationship but not necessarily in a long-term one. The reason is that in the former relational contracts are never feasible.

Valsecchi (1996) shows that appropriate job design can restrict the set of sequential equilibria to the Pareto optimal one if tasks are performed sequentially in a team production process. Through appropriate task grouping, the principal can exploit workers' private information about their own effort or the effort exerted by their colleagues. As in my framework, it is not possible to measure effort in each single task. However, the model is static and thus does not consider the use of non-verifiable information.

In the next section, the model is introduced. In section 3, I derive the optimal combination of implicit and explicit contracts for a given set of tasks. Section 4 analyzes the question of *when* tasks should be split between agents, while the section 5 examines *how* tasks should be grouped into jobs. Section 6 generalizes the results to an arbitrary number of tasks and agents and discusses the model assumptions. The last section concludes.

2 The model

I consider a relationship between a principal and one or two agents. All parties are risk-neutral. The principal is the owner of the firm in which the agents can be

employed. In each period, the probability that the principal-agent(s) relationship will be repeated in the following period is exogenously given by $\rho \in (0, 1)$.

There are three tasks that jointly affect firm value Y . Y is either high or low, $Y \in \{0, 1\}$, and is realized at the end of each period. I define $N := \{1, 2, 3\}$ as the set of tasks and $e_t \geq 0$ as the non-observable effort exerted in task $t \in N$. Furthermore, e denotes the vector of all efforts, $e = (e_1, e_2, e_3)^T$.⁸ The probability that firm value is high given e is assumed to be

$$\text{prob}[Y = 1|e] = \min\{f^T e, 1\} = \min\{f_1 e_1 + f_2 e_2 + f_3 e_3, 1\}, \quad (1)$$

where $f \in \mathbb{R}^3$ and $f_t > 0$ for all $t \in N$, i.e., all tasks are productive.

We could Y interpret alternatively as the value of a division or department in which the agents are employed. Then, the realization of Y would not only depend on the effort in the three tasks under consideration. For example, none of the following results would change if we had $\text{prob}[Y = 1|e] = \min\{y + f^T e, 1\}$, where y is determined by the contribution of other employees and is independent of e .

The realization of Y is observed by the principal and all employed agents but is non-verifiable. However, there is a verifiable performance measure $P \in \{0, 1\}$ that is also realized at the end of each period, where

$$\text{prob}[P = 1|e] = \min\{g^T e, 1\}, \quad (2)$$

$g \in \mathbb{R}^3$, $g_t > 0$ for all $t \in N$.⁹ Similar to Y , the realization of P could also depend on the performance of other employees. Given f, g , and e , the realizations of Y and P are independent.

The principal cannot perform any task from N herself. She can hire either one or two homogeneous agents to perform the tasks. If she employs two agents, she must also decide how to group tasks into jobs. I assume that each task can be assigned to

⁸All vectors are column vectors. Superscript T denotes transpose.

⁹Similar multi-tasking approaches are widely-used in the literature, see, e.g., Feltham and Xie (1994), Datar, Kulp, and Lambert (2001), and Baker (2002).

one agent only. Furthermore, a task assignment has to be maintained in all future periods. This also implies that the initially chosen number of agents is invariant over time.¹⁰

An agent's non-observable cost of exerting effort e is

$$c(e) = \frac{c}{2}e^Te, \quad c > 0, \quad (3)$$

i.e., $c(e)$ is separable and quadratic. Agents have unlimited liability. Their opportunity costs of working for the principal are zero in each period. Thus, there are no a priori costs of employing two rather than one agent. How results are affected by positive opportunity costs is discussed in section 6.

For simplicity, I assume that f, g , and c are such that the probabilities f^Te and g^Te are always smaller than one at the optimal (first- and second-best) solution.¹¹ The vector of first-best efforts, denoted by e^{FB} , maximizes expected firm value minus costs of effort, i.e.,

$$e^{FB} = \operatorname{argmax}_e f^Te - \frac{c}{2}e^Te. \quad (4)$$

Thus, $e^{FB} = c^{-1}f$ leading to an expected profit of $\frac{f^Tf}{2c}$.

Timing is as follows in each period: At the beginning of the period, the principal individually offers each agent an explicit wage contract specifying some guaranteed fixed payment and an explicit bonus that will be paid at the end of the period if $P = 1$. Additionally, the principal may offer an implicit bonus that he promises to pay at the end of the period if $Y = 1$. However, since Y is non-verifiable, an agent will rely on such a promise only if he believes that it is in the principal's interest not to renege on it. Given the explicit and the relational contract, each agent chooses his effort level(s). Afterwards, Y and P are realized and each agent is rewarded

¹⁰This can be justified if, for example, agents have to learn how to perform a task before production can take place. Then, changing the number of agents in future periods would lead to additional learning costs for at least one task. Such costs are often at least partly borne by the firm and might be prohibitively high. I will discuss the impact of this assumption in section 6.

¹¹It can be shown that this is the case if $\max\{f^Tf, f^Tg\} < c$.

according to his explicit contract. If $Y = 1$, the principal decides whether to pay the implicit bonuses to one or both agents.

In the remainder of this section and in the following one I analyze the case in which the principal employs only one agent who performs all three tasks. First assume that the agent does not trust the principal to pay any bonus based on the realization of Y .¹² Let α (fixum) and β (bonus) denote the components of the explicit wage contract that the principal offers to the agent. Then, the principal's optimization problem is

$$\max_{\alpha, \beta, e} f^T e - (\alpha + \beta g^T e), \quad (5)$$

$$\text{s.t. } e = \operatorname{argmax}_{\hat{e}} \alpha + \beta g^T \hat{e} - \frac{c}{2} \hat{e}^T \hat{e}, \quad (6)$$

$$0 \leq \alpha + \beta g^T e - \frac{c}{2} e^T e. \quad (7)$$

This problem has already been analyzed in similar forms, e.g., by Baker (2002) and Gibbons (2005).¹³ From the incentive compatibility constraint (6), the agent exerts efforts $e(\beta) = \frac{\beta}{c} g$. For each β , the principal will choose α so that the agent's participation constraint (7) is binding. Thus, the optimal bonus $\tilde{\beta}$ maximizes $f^T e(\beta) - \frac{c}{2} (e(\beta))^T e(\beta)$, i.e.,

$$\tilde{\beta} = \frac{f^T g}{g^T g} = \frac{\|f\|}{\|g\|} \cos \theta, \quad (8)$$

where

$$\|f\| := \sqrt{f_1^2 + f_2^2 + f_3^2} \quad \text{and} \quad \cos \theta := \frac{f^T g}{\|f\| \|g\|}. \quad (9)$$

By these definitions, $\|f\|$ and $\|g\|$ are the lengths of the vectors f and g , respectively, and θ is the angle between them. Because all components of f and g are assumed to

¹²The circumstances under which this happens are discussed in the next section.

¹³In both papers, firm value is $f^T e$ plus a noise term, and the performance measure is $g^T e$ plus another noise term. In Baker (2002) the agent is also risk-averse.

be positive, $\cos \theta$ is also positive.¹⁴ The resulting expected profit for the principal is

$$\tilde{\pi} = \frac{\|f\|^2}{2c} \cos^2 \theta. \quad (10)$$

As pointed out by Baker (2002) and Gibbons (2005), there are two features that determine the optimal explicit bonus $\tilde{\beta}$: scaling, as given by $\|f\|/\|g\|$, and alignment, as given by $\cos \theta$. $\cos \theta$ can be interpreted as a measure of alignment (or congruity) between firm value and performance measure. The higher $\cos \theta$ the better aligned are f and g and, thus, the more useful is the performance measure for efficiently directing effort to the different tasks. Therefore, the optimal explicit bonus and expected profit increase in $\cos \theta$.

If $\cos \theta = 1$, i.e., f and g are perfectly aligned, the first-best solution is implemented by scaling the bonus appropriately. Thus, scaling only corrects for the difference in the lengths of f and g . For instance, if $\cos \theta = 1$ but firm value is more sensitive to changes in effort than the performance measure, i.e., $\|f\| > \|g\|$, we have $\tilde{\beta} > 1$.

I henceforth assume that the principal cannot implement first-best efforts by a pure explicit contract, i.e., $\cos \theta < 1$. I analyze the optimal combination of explicit and relational contracts in the next section.

3 Combining explicit and relational contracts

The analysis in this section is similar to the one in Baker et al. (1994). The main difference is that these authors consider the case of a single agent performing only one task. The productivity of his effort with respect to the contractible performance measure is observed only by the agent after the explicit contract has been signed.

¹⁴This implies that the performance measure is never completely useless. Allowing for zero components in f and g does not affect the results, but leads to tedious case distinctions under task splitting.

In general, this productivity is different from the effort's true contribution to firm value. This creates a congruity problem similar to the one considered here.¹⁵

Assume the principal offers the agent an explicit contract as described in the foregoing section. Additionally, suppose that she can credibly promise to pay an implicit bonus γ if $Y = 1$. Then the agent chooses $e(\beta, \gamma)$ to solve the problem

$$\max_e \alpha + \beta g^T e + \gamma f^T e - \frac{c}{2} e^T e, \quad (11)$$

i.e.,

$$e(\beta, \gamma) = \frac{1}{c}(\beta g + \gamma f). \quad (12)$$

For each combination of β and γ , the principal sets α so that the agent's participation constraint

$$\alpha + \beta g^T e(\beta, \gamma) + \gamma f^T e(\beta, \gamma) - \frac{c}{2} (e(\beta, \gamma))^T e(\beta, \gamma) \geq 0 \quad (13)$$

is binding.

When determining the optimal combination of explicit and implicit bonus, the principal must take into account that her promise to pay γ if firm value is high must be trustworthy to the agent. To model the role of trust, I assume that if the principal once reneges on the relational contract, the agent will never trust her again to pay an implicit bonus. Thus, if the principal breaks the relational contract, his fallback position is a pure explicit contract leading to profit $\tilde{\pi}$ (see (10)) in all future periods. Therefore, the principal chooses β and γ to solve the problem

$$\max_{\beta, \gamma} f^T e(\beta, \gamma) - \frac{c}{2} e(\beta, \gamma)^T e(\beta, \gamma) \quad (14)$$

$$\text{s.t. } \gamma \leq \sum_{t=1}^{\infty} \rho^t \left[f^T e(\beta, \gamma) - \frac{c}{2} e(\beta, \gamma)^T e(\beta, \gamma) - \tilde{\pi} \right]. \quad (15)$$

Inequality (15) is the principal's commitment constraint. It says that the principal's short-term profit from reneging on the relational contract, γ , must not exceed

¹⁵Furthermore, in Baker et al. (1994), the agent's opportunity costs are positive so that the principal's profit under a pure explicit contract can be negative.

the associated expected long-term loss, which is given by the term on the right-hand side of (15). If (15) was not satisfied, the agent would anticipate that the principal will not stick to the informal agreement if high firm value is realized.

When solving the principal's problem as given by (14) and (15), we get from the first-order condition for the optimal explicit bonus that

$$\beta(\gamma) = (1 - \gamma) \frac{\|f\|}{\|g\|} \cos \theta = (1 - \gamma) \tilde{\beta}. \quad (16)$$

Then the principal's expected profit can be written as

$$\pi(\gamma) = \frac{\|f\|^2}{2c} \cos^2 \theta + \frac{\gamma(2 - \gamma)}{2c} \|f\|^2 (1 - \cos^2 \theta). \quad (17)$$

If γ is credible, expected profit per period increases compared to a pure explicit contract by the second term on the right-hand side of (17). This term strictly increases in γ and is maximal at $\gamma = 1$. If $\gamma = 1$, the agent becomes the residual claimant and, thus, exerts first-best effort in each task. In return, he pays the expected profit to the principal, i.e., $\alpha = -\frac{\|f\|}{2c}$.

However, the principal will in general not be able to commit to an implicit bonus of $\gamma = 1$. After substituting β , her commitment constraint (15) becomes

$$\phi\gamma \leq \frac{\gamma(2 - \gamma)}{2c} \|f\|^2 (1 - \cos^2 \theta), \quad (18)$$

where $\phi := (1 - \rho)/\rho$.

The principal chooses the highest $\gamma \in [0, 1]$ which satisfies (18) so that the optimal implicit bonus is

$$\gamma^* = \begin{cases} 1 & \text{if } 2c\phi \leq \|f\|^2 (1 - \cos^2 \theta) \\ 2 \left(1 - \frac{c\phi}{\|f\|^2 (1 - \cos^2 \theta)} \right) & \text{if } c\phi < \|f\|^2 (1 - \cos^2 \theta) < 2c\phi \\ 0 & \text{if } \|f\|^2 (1 - \cos^2 \theta) \leq c\phi \end{cases} \quad (19)$$

The principal's expected profit is

$$\pi(\gamma^*) = \begin{cases} \frac{\|f\|^2}{2c} & \text{if } \gamma^* = 1 \\ \frac{\|f\|^2}{2c} \cos^2 \theta + 2\phi \left(1 - \frac{c\phi}{\|f\|^2 (1 - \cos^2 \theta)} \right) & \text{if } 0 < \gamma^* < 1 \\ \frac{\|f\|^2}{2c} \cos^2 \theta & \text{if } \gamma^* = 0 \end{cases} \quad (20)$$

The principal can commit to a high implicit bonus if her loss from breaking the relational contract is large. The per-period loss from reneging on the relational contract, which is given on the right-hand side of (18), increases in the term $\|f\|^2(1 - \cos^2 \theta)$. Thus, γ^* also increases in this term. Given $\cos^2 \theta$, a high value of $\|f\|$ means that expected firm value strongly responds to changes in effort. Therefore, the benefit from better aligning incentives by paying an implicit bonus contingent on firm value Y is large. Given $\|f\|$, a low value of $\cos^2 \theta$, i.e., a strongly distorted performance measure, also makes the use of implicit incentives more desirable. This leads to the first proposition.

Proposition 1 *Relational contracts exist in environments where well aligned performance measures are not available¹⁶ or firm value is highly responsive to changes in effort.*

Furthermore, the optimal implicit bonus increases in ρ and decreases in c . The intuition is straightforward. A high probability that the principal-agent relationship will continue increases the expected loss from breaking the relational contract. Low effort costs increase the per-period benefit from using a relational contract (see (18)) and, therefore, also the loss from reneging on it.

In figure 1, the optimal implicit bonus and the resulting profit are depicted for fixed $\|f\|$ and varying $\cos^2 \theta$. The higher $\cos^2 \theta$ the less distorted the performance measure, and, therefore, the smaller is the principal's loss from reneging on the relational contract. Thus, as already explained above, the maximal feasible implicit bonus decreases in $\cos^2 \theta$. As a result, the principal benefits from a less distorted performance measure if and only if she is not able to pay an implicit bonus.

On the other hand, holding $\cos^2 \theta$ constant while increasing $\|f\|$ fixes the distortion of the performance measure but increases each tasks' (expected) marginal

¹⁶In their framework, Baker et al. (1994) derive the same result.

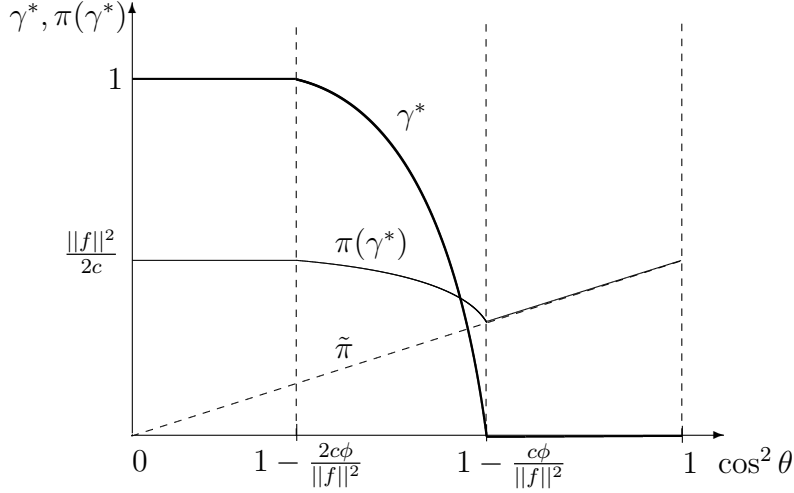


Figure 1: Optimal implicit bonus and expected profit.

productivity by the same factor. This means that firm value becomes more responsive to changes in effort. Therefore, both γ^* and $\pi(\gamma^*)$ increase in $\|f\|$.

I summarize these results in the following proposition.

Proposition 2 (i) For fixed $\|f\|$, the optimal implicit bonus decreases in $\cos \theta$. The principal's expected profit strictly increases in $\cos \theta$ if and only if she cannot commit to an implicit bonus, i.e., if $\cos^2 \theta \geq 1 - c\phi/\|f\|^2$. (ii) For fixed $\cos \theta$, the optimal implicit bonus and expected profit increase in $\|f\|$.

4 When should tasks be split?

In the previous section, I have focussed on the case of one agent. In this section, I analyze under which circumstances the principal benefits from splitting tasks between two agents. Obviously, there is no use of task splitting if the principal can set first-best incentives when employing one agent, i.e., if $\gamma^* = 1$. Therefore, I henceforth consider the case $\gamma^* < 1$.

Assume that task i can be performed by another agent, while tasks j and k are non-separable, where $\{i, j, k\} = N$, $j < k$. If there is task splitting, agent 1 performs

task i and agent 2 performs tasks j and k . I define

$$e^{-i} := (e_j, e_k)^T, \quad f^{-i} := (f_j, f_k)^T, \quad g^{-i} := (g_j, g_k)^T, \quad (21)$$

and

$$\cos \theta_{-i} := \frac{(f^{-i})^T g^{-i}}{\|f^{-i}\| \|g^{-i}\|}. \quad (22)$$

Furthermore, let β_i, γ_i and β_{-i}, γ_{-i} denote the explicit and implicit bonus for agent 1 and agent 2, respectively. Analogously, α_i and α_{-i} denote the fixed payments.

I assume that if the principal breaks a relational contract with one agent, both agents will not rely on relational contracts in all future periods. This implies that an agent can observe whether or not the principal kept an implicit agreement with his colleague.¹⁷ Furthermore, I assume that agent 1 observes the explicit and relational contract offered to agent 2 and vice versa.

Suppose that implicit bonuses are credible. Then, given the effort levels of agent 2, e^{-i} , agent 1 chooses e_i to solve the problem

$$\max_{e_i} \alpha_i + \beta_i g^T e + \gamma_i f^T e - \frac{c}{2} e_i^2. \quad (23)$$

Analogously, given the effort of agent 1, e_i , agent 2 chooses e^{-i} to solve

$$\max_{e^{-i}} \alpha_{-i} + \beta_{-i} g^T e + \gamma_{-i} f^T e - \frac{c}{2} (e^{-i})^T e^{-i}. \quad (24)$$

It follows that

$$e_i(\beta_i, \gamma_i) = \frac{1}{c}(\beta_i g_i + \gamma_i f_i), \quad e^{-i}(\beta_{-i}, \gamma_{-i}) = \frac{1}{c}(\beta_{-i} g^{-i} + \gamma_{-i} f^{-i}), \quad (25)$$

i.e., an agent's effort choice does not depend on the effort choice of his colleague. However, the joint effort of both agents determines the probabilities of high firm

¹⁷I make this assumption because it is prevalent in the literature (see, e.g., Bull (1987)). However, in the case of three tasks and two agents, it can be shown that the results do not change when only the agent who was cheated does no longer rely on relational contracts. I will discuss the impact of this assumption in more general settings in section 6.

value and a favorable performance measure and, therefore, also the expected payment to each agent. Thus, when deciding whether to accept the contract offered by the principal, each agent must anticipate the effort choice of his colleague. Moreover, each agent can trust the principal to pay his individual bonus only if the principal finds it beneficial to pay both implicit bonuses simultaneously. Thus, if an agent does not know the implicit bonus offered to his colleague, he cannot judge the credibility of the promise that the principal made to him. Therefore, the assumption that each agent observes the contract offered to his colleague simplifies the analysis.¹⁸

Let γ_i^* and γ_{-i}^* denote the optimal implicit bonuses that are paid to agent 1 and agent 2, respectively. As in the previous section, γ^* denotes the optimal implicit bonus with one agent performing all tasks alone. In order to compare the principal's profit under task splitting with her profit when all tasks are performed by one agent, I first derive the following proposition.

Proposition 3 *Assume that $\gamma^* < 1$ and task i is assigned to agent 1. Then agent 1 exerts effort e_i^{FB} and $\gamma_i^* = 0$. Furthermore, $\gamma_{-i}^* \leq \gamma^*$ and agent 2 exerts $(e^{FB})^{-i}$ if and only if $\cos \theta_{-i} = 1$. The principal's profit is*

$$\pi^S(\gamma_{-i}^*) = \begin{cases} \frac{f_i^2}{2c} + \frac{\|f^{-i}\|^2}{2c} \cos^2 \theta_{-i} + 2\phi \left(1 - \frac{c\phi}{\|f^{-i}\|^2(1-\cos^2 \theta_{-i})} \right) & \text{if } 0 < \gamma_{-i}^* < 1 \\ \frac{f_i^2}{2c} + \frac{\|f^{-i}\|^2}{2c} \cos^2 \theta_{-i} & \text{if } \gamma_{-i}^* = 0 \end{cases}$$

Proof See appendix.

Proposition 3 says that the principal provides first-best incentives for agent 1 by a pure explicit contract. Furthermore, an agent performing two tasks always receives a lower implicit bonus than an agent who performs three tasks.

When determining the optimal combination of an explicit and relational contract for agent 2, the principal can proceed as if he would solve the problem

$$\max_{\beta, \gamma} (f^{-i})^T e^{-i}(\beta, \gamma) - \frac{c}{2} (e^{-i}(\beta, \gamma))^T e^{-i}(\beta, \gamma) \quad (26)$$

¹⁸In the case of three tasks this assumption can be dropped. As we will see, agent 1 exerts first-best effort in his task under a pure explicit contract. Thus, by knowing f and g and the tasks assigned to himself, each agent can anticipate which contract will be offered to his colleague.

$$\text{s.t. } \phi\gamma \leq (f^{-i})^T e^{-i}(\beta, \gamma) - \frac{c}{2}(e^{-i}(\beta, \gamma))^T e^{-i}(\beta, \gamma) - \frac{\|f^{-i}\|}{2c} \cos^2 \theta_{-i}, \quad (27)$$

which is equivalent to the problem analyzed in section 3. This is due to two reasons: Since agent 1 is responsible for only one task, he cannot misallocate effort between tasks. Consequently, there is no need to pay an implicit bonus to agent 1. Second, an agent's effort choice affects his colleague only in terms of his expected payment. This does not cause any problems, because the principal can always make agents' participation constraints binding by individually adjusting the fixed payments α_i and α_{-i} .

In the proof of proposition 3 it is shown that

$$\|f^{-i}\|^2(1 - \cos^2 \theta_{-i}) \leq \|f\|^2(1 - \cos^2 \theta), \quad (28)$$

and therefore, by (19), $\gamma_{-i}^* \leq \gamma^*$. That is, if the principal withdraws an arbitrary task from an agent, the implicit bonus she can commit to paying to this agent decreases. The intuition for this result is as follows. Withdrawing a task from an agent affects his implicit bonus in two different ways. First, his performance becomes less important for the firm value because $\|f^{-i}\| < \|f\|$. Second, the congruency problem associated with this agent may become more or less severe, i.e., $\cos \theta_{-i}$ can be smaller or larger than $\cos \theta$.¹⁹ The first effect decreases the agent's maximal feasible implicit bonus. The second effect works in the opposite direction if f^{-i} and g^{-i} are worse aligned than f and g . However, the first effect always dominates.

As a result, if $\gamma^* < 1$, first-best efforts can be implemented for agent 2 if and only if this is possible through a pure explicit contract, i.e., if f^{-i} and g^{-i} are perfectly aligned.

Although task splitting always leads to first-best incentives for one task, the next proposition shows that it is often optimal to assign all tasks to one agent.

¹⁹At first sight one might think that $\cos \theta_{-i} \geq \cos \theta$, i.e., the congruency problem is always less severe with only two tasks. However, consider, e.g., $f = (1, 0, x)^T$ and $g = (0, 1, y)^T$. In this case, $\cos \theta_{-3} < \cos \theta$ for all $x, y > 0$.

Proposition 4 (i) If $\gamma^* = 0$, the principal prefers task splitting. (ii) If $\gamma_{-i}^* > 0$, the principal prefers to assign all tasks to one agent.

Proof See appendix.

By proposition 4, the principal prefers task splitting if relational contracts are infeasible no matter whether she employs one or two agents. However, if an agent who performs two tasks receives an implicit bonus, expected profit increases if the third task is also assigned to this agent.

To understand the intuition for these results, note that (28) is equivalent to

$$\|f\|^2 \cos^2 \theta \leq f_i^2 + \|f^{-i}\|^2 \cos^2 \theta_{-i}. \quad (29)$$

From this inequality it follows immediately that the expected profit under pure explicit contracts is always larger under task splitting, i.e., $\pi(0) \leq \pi^S(0)$. The reason is that even if f^{-i} and g^{-i} are worse aligned than f and g , the misalignment between f^{-i} and g^{-i} cannot become so large that it dominates the positive effect of setting first-best incentives for task i . Thus, we obtain result (i). Furthermore, it is clear that $\pi(\gamma^*) = \pi^S(\gamma_{-i}^*)$ if (28) binds.²⁰

However, if (28) does not bind and $\gamma_{-i}^* > 0$, the increase in expected profits due to the use of an implicit bonus is larger when all tasks are performed by one agent, i.e.,

$$2\phi \left(1 - \frac{c\phi}{\|f^{-i}\|^2(1 - \cos^2 \theta_{-i})} \right) < 2\phi \left(1 - \frac{c\phi}{\|f\|^2(1 - \cos^2 \theta)} \right). \quad (30)$$

This is due to the fact that an agent who performs three tasks receives a higher implicit bonus. Therefore, as long as $\gamma_{-i}^* > 0$, the overall improvement of explicit contracts under task splitting never outweighs the loss due to a weakened relational contract for agent 2.

Only if f^{-i} and g^{-i} are so well aligned that the principal cannot commit to an implicit bonus for agent 2, pure explicit contracts under task splitting may dominate

²⁰This happens in the special case of $g_i/f_i = (g_j^2 + g_k^2)/(f_j g_j + f_k g_k)$ as can be seen from the proof of proposition 3.

a combination of pure and relational contracts when tasks are not split. This is the only case that is not considered in proposition 4, namely, $\gamma^* > 0$ and $\gamma_{-i}^* = 0$. In this case, task splitting is optimal if f^{-i} and g^{-i} are sufficiently well aligned because then incentives with pure explicit contracts are close to first-best under task splitting. In the extreme case of $\cos \theta_{-i} = 1$, the principal implements first-best efforts if she employs two agents.

Formally, if $\gamma^* > 0$ and $\gamma_{-i}^* = 0$, it can be easily verified that assigning task i to another agent leads to a higher expected profit if

$$\|f^{-i}\|^2(1 - \cos^2 \theta_{-i}) \leq \|f\|^2(1 - \cos^2 \theta) - 4c\phi \left(1 - \frac{c\phi}{\|f\|^2(1 - \cos^2 \theta)}\right). \quad (31)$$

The right-hand side of this inequality decreases in $\|f\|^2(1 - \cos^2 \theta)$ and increases in ϕ .²¹ Thus, inequality (31) is likely to hold if $\|f\|^2(1 - \cos^2 \theta)$ is small and ϕ is large (i.e., γ^* is small), and/or if $\|f^{-i}\|^2(1 - \cos^2 \theta_{-i})$ is small (i.e., the congruency problem for agent 2 is not severe).

By combining propositions 1 and 4, we obtain that all tasks should be assigned to one agent if (I) the performance measure is not suitable to provide incentives for the two-task job (i.e., θ_{-i} is large) or (II) firm value strongly responds to effort changes in the two-task job. More loosely speaking, the principal should not split tasks if a pure explicit contract performs badly in the two-task job.

Under certain conditions, pure explicit contracts can induce production workers to allocate effort efficiently across tasks. For instance, Lazear (2000) shows that piece rates combined with some form of quality control can provide workers with incentives to produce high output without neglecting quality. If production workers can be closely monitored, even the number of hours worked may serve as a good proxy for performance. Usually, the output of supervisors and managers is less concrete, and, therefore, more difficult to measure. Thus, (I) suggests that jobs tend to consist of more tasks on higher hierarchy levels.

²¹This is because $c\phi < \|f\|^2(1 - \cos^2 \theta) < 2c\phi$ since $0 < \gamma^* < 1$.

Furthermore, (II) implies that job assignments consisting of a broad range of tasks are more likely if these tasks strongly affect firm value. Suppose firm value Y depends on production and management tasks as explained in section 2. Then, (II) also leads to the conclusion that management tasks are more likely to be assigned to one agent than production tasks, because management tasks generally affect firm value more strongly.

Finally, since implicit bonuses also increase in the probability that the principal-agent relationship continues, employees that are more likely to stay with the firm should perform more tasks.

5 How should tasks be split?

In this section, I assume that all three tasks are separable and must be split between two agents. For example, due to lack of time, it might not be possible that one agent performs all tasks. Then, the question arises which task the principal should assign to agent 1, i.e., to the agent who is responsible for only one task.

There are two cases in which first-best effort is implemented in each task. First, if there is a task t such that $\cos \theta_{-t} = 1$, assigning this task to agent 1 is optimal. Then, by proposition 3, the principal induces first-best effort in each task by pure explicit contracts. Second, if it is possible to have $\gamma_{-t}^* = 1$ for some task t , assigning this task to agent 1 also leads to first-best incentives. Agent 2 then exerts first-best efforts under a pure relational contract.

Now assume the principal cannot implement first-best efforts under task splitting. Furthermore, I denote the task t for which f_t is maximal (minimal) as the most (least) important task. Let again task i be the task that is assigned to agent 1.

First consider the case that it is never possible to pay an implicit bonus to agent 2, i.e., $\gamma_{-t}^* = 0$ for all t . Then, by proposition 3, if agent 1 performs task i , the

principal's expected profit is

$$\frac{\|f\|^2}{2c} - \frac{\|f^{-i}\|^2}{2c}(1 - \cos^2 \theta_{-i}). \quad (32)$$

Thus, expected profit decreases in $\|f^{-i}\|(1 - \cos^2 \theta_{-i})$.

If $0 < \gamma_{-t}^* < 1$ for all t , expected profit is

$$\frac{\|f\|^2}{2c} - \frac{\|f^{-i}\|^2}{2c}(1 - \cos^2 \theta_{-i}) + 2\phi \left(1 - \frac{c\phi}{\|f^{-i}\|^2(1 - \cos^2 \theta_{-i})} \right). \quad (33)$$

In contrast to (32), (33) strictly increases in $\|f^{-i}\|(1 - \cos^2 \theta_{-i})$.²² We therefore get the following result.

Proposition 5 (i) *If $\gamma_{-t}^* = 0$ for all t , agent 1 should perform the task i which satisfies*

$$i = \operatorname{argmin}_{t \in N} \|f^{-t}\|(1 - \cos^2 \theta_{-t}). \quad (34)$$

(ii) *If $0 < \gamma_{-t}^* < 1$ for all t , agent 1 should perform the task i which satisfies*

$$i = \operatorname{argmax}_{t \in N} \|f^{-t}\|(1 - \cos^2 \theta_{-t}). \quad (35)$$

Although performance is first-best in the one-task job, assigning the most important task to agent 1 is in general not optimal. An exception is the case in which agent 2 never receives an implicit bonus and $\cos \theta_{-t}$ is independent of t . Usually, the latter condition does not hold. Then it will not be optimal to assign the most important task to agent 1 if the corresponding f^{-t} and g^{-t} are too badly aligned. In the special case in which all tasks are equally important, i.e., $f_1 = f_2 = f_3$, tasks should be assigned so that f^{-i} and g^{-i} are best aligned.

If agent 2 always receives an implicit bonus and $\cos \theta_{-t}$ is independent of t , it is even optimal to assign the least important task to agent 1. The reason is that, the more important the tasks assigned to agent 2, the larger is the principal's loss if she reneges on the relational contract with this agent. Therefore, the maximal feasible

²²This is due to the fact that $\|f^{-i}\|(1 - \cos^2 \theta_{-i}) < 2c\phi$ because $\gamma_{-i}^* < 1$.

implicit bonus for agent 2 increases in the importance of the tasks performed by this agent. Moreover, a higher implicit bonus increases expected profit more strongly than having first-best effort in the most important task.

In the other extreme case, if agent 2 always receives an implicit bonus but $f_1 = f_2 = f_3$, tasks should be assigned so that f^{-i} and g^{-i} are worst aligned since the optimal implicit bonus also increases in misalignment.

If the optimal implicit bonus for agent 2 is not always either zero or positive under each possible task assignment, the improvement in explicit contracts (if $\gamma_{-i}^* = 0$) must be traded off against the benefit from having a relational contract with agent 2 (if $\gamma_{-i}^* > 0$). Optimal task splitting then depends on the particular form of f and g .

6 Discussion

In this section, I discuss the generalization of the analysis to more tasks and agents as well as some of the model assumptions. I also give some directions for further research.

The analysis of sections 3 and 4 can, under some restrictions, be generalized to the case of splitting n tasks between l agents, where $l < n$. Clearly, all results in section 3 apply for any arbitrary number of tasks performed by a single agent.

However, the analysis in section 4 becomes more complicated if the number of tasks increases. If there are, for instance, four tasks and two agents, it may be optimal to pay both agents an implicit bonus. Under the assumption that both agents lose trust if the principal reneges on a relational contract with one of them, the principal's only commitment constraint is

$$\phi(\gamma_i + \gamma_{-i}) \leq f^T e - \sum_{l=i,-i} (\alpha_l + \beta_l g^T e + \gamma_l f^T e) - \bar{\pi}, \quad (36)$$

where $\bar{\pi}$ denotes the profit under pure explicit contracts.²³ Naturally, the optimal

²³Compare constraint (48) in the appendix. Of course, all vectors are now four-dimensional.

implicit bonuses cannot be determined independently of each other. This means, in particular, that the derivation of the optimal contract for one agent cannot be boiled down to the problem analyzed in section 3 by just dropping the tasks performed by the other agent. This feature greatly simplified the analysis in the case of three tasks. However, it can be reestablished by changing the modelling of trust.

Assume that if the principal reneges on one relational contract only the agent who was cheated loses trust. This assumption leads to additional commitment constraints for the principal and, therefore, limits the set of implementable implicit bonuses relative to (36). Then it can be shown that agents' optimal contracts are independent of each other and all results of section 4 can be extended to the case of splitting n tasks between l agents.²⁴ In particular, assigning all tasks to one agent will be optimal if *at least* one agent receives an implicit bonus under each arbitrary task splitting. If, on the other hand, it is not credible to promise an implicit bonus to an agent who performs all tasks, task splitting always increases profits.

While I was not able to derive clear-cut results for the general optimal task assignment under the initial modelling of trust, it is clear how results will change relative to the case just described. If, under task splitting, *all* agents lose trust when the principal reneges on only one relational contract, the temptation to renege is smaller. As explained above, this results in a larger set of implementable relational contracts. Thus, task splitting will more frequently be preferred to assigning all tasks to one agent.

Given that tasks must be split, the optimal grouping of tasks into jobs depends on the particular form of f and g and the number of agents. Therefore, the results of section 5 more generally apply only for the splitting of n tasks between $n - 1$ agents. Furthermore, if there is only the possibility of withdrawing one task from a particular agent, the results explain which one the principal should choose.

I made the assumption that the principal cannot change the task assignment in

²⁴Proofs are available from the author upon request.

future periods. This affects her fallback position after breaking implicit agreements and, therefore, may be critical for the results derived. First consider the case in which the principal initially employs two agents and agrees on a positive implicit bonus with agent 2. If the principal reneged on the implicit agreement, she would not want to dismiss one agent because task splitting is superior under pure explicit contracts.

However, if there is initially a single agent performing all tasks, the principal would benefit from splitting tasks after breaking the relational contract. Thus, in this case the assumption of inflexible job design matters. It worsens the fallback position of the principal and, therefore, leads to a higher feasible implicit bonus for the single agent. Hence, ex ante it is in the principal's interest to commit to not splitting tasks in the future. I assumed that such a commitment is possible because the costs of hiring another agent (e.g., learning costs) are higher than the benefits.²⁵ However, if such a commitment is not possible, assigning all tasks to a single agent will be less often preferred.

Furthermore, I assumed that agents' reservation utility is zero. Now assume that an agent's alternative wage per period is $\bar{w} > 0$, where $\tilde{\pi} - 2\bar{w} > 0$, i.e., the expected profit under pure explicit contracts is still positive. Then it is easily verified that the principal's expected profit is $\pi(\gamma^*) - \bar{w}$ if all tasks are performed by one agent, and $\pi^S(\gamma_{-i}^*) - 2\bar{w}$ under task splitting.²⁶ Thus, task splitting becomes less attractive than with alternative wages of zero. At the end of section 4, I argued that jobs should be more complex on higher hierarchy levels. This conclusion is strengthened by positive but not too high opportunity costs since alternative wages will, in general, increase in the hierarchy level.

²⁵Explicitly, I assume that $K \geq \frac{f_i^2}{2c} + \frac{\|f_{-i}\|^2}{2c} \cos^2 \theta_{-i} - \frac{\|f\|^2}{2c} \cos^2 \theta$, where K denotes the costs of learning how to perform a task which have to be borne by the firm, e.g., trainee programs or opportunity costs from having senior staff to teach the new colleague.

²⁶The analysis becomes more complicated if $\tilde{\pi} - 2\bar{w} < 0$ because then optimal explicit and implicit bonuses depend on \bar{w} . Examining this problem may be subject to future research.

In this paper, I exclusively focussed on optimal job design under congruency problems. Of course, as the literature survey in the introduction shows, there are many other factors that influence the optimal assignment of tasks within an organization. In particular, there might be complementarities or substitutabilities between tasks. Consider, for example, the cost function

$$C(e_1, e_2, e_3) = c \sum_{i=1}^3 e_i^2 + c\delta e_1 e_2 e_3, \quad \delta \in \mathbb{R}. \quad (37)$$

Then, if $\delta < 0$ ($\delta > 0$), all tasks are complements (substitutes). If δ approaches zero, we come close to the case of independent tasks and the results derived in this paper apply. Presumably, if $\delta < 0$ ($\delta > 0$), task splitting becomes less (more) preferable. The cost function could also be extended to the case where some tasks are complements and others substitutes. The analysis of these problems may be subject to future research.

7 Conclusion

In this paper, I derived two main results concerning the optimal interplay between job design and relational contracts.

First, if the principal cannot commit to paying an implicit bonus to an agent who performs all tasks, the principal is better off by splitting tasks because this improves the performance of explicit contracts. This case occurs when the objective performance measure is not strongly distorted, or when firm value does not strongly respond to effort changes in the given set of tasks. Then, the principal's loss from reneging on a relational contract is small, so that she is not able to commit to paying an implicit bonus.

Second, the principal prefers not to split tasks whenever it is possible to pay an implicit bonus under task splitting. The reason is that, if an agent performing a given set of tasks receives an implicit bonus, assigning an additional task to this

agent allows the principal to commit to an even higher implicit bonus. This is due to the fact that the principal's loss from breaking a relational contract increases in the number of tasks that an agent performs. The strengthened relational contract always outweighs the loss from not having first-best effort in the additional task.

Overall, broad task assignments are optimal if objective performance measurement is difficult, or firm value is highly responsive to effort changes in the given tasks. This implies that task assignments tend to be more complex on higher hierarchy levels within a firm.

I assumed that there is only one exogenously given contractible performance measure. In many situations, the principal can invest in generating additional performance measures, thereby improving the performance of explicit contracts. However, doing so increases the costs of performance measurement. The analysis in this paper shows that job design can be a substitute to generating performance measures. When there is a second performance measure for three tasks, first-best incentives can be implemented for each tasks under task splitting.²⁷ However, instead of incurring costs for better objective performance measurement, the principal might prefer to assign all tasks to one agent if this leads to a high-powered relational contract.

Appendix

Proof of proposition 3. First consider the principal's fallback position when she reneges on one or both relational contracts. In this case, she will offer in each

²⁷The two performance measures must be linearly independent. That is, if the performance measures are characterized by the vectors g and h , we must have $g \neq \lambda h$ for all $\lambda \in \mathbb{R}$. Then, the principal can always implement first-best incentives in the two-task job if she appropriately weights the two performance measures in a pure explicit contract.

following period the explicit contracts that solve the problem

$$\max_{\substack{\alpha_l, \beta_l, e_l \\ l=i, -i}} f^T e - \sum_{l=i, -i} (\alpha_l + \beta_l g^T e), \quad (38)$$

$$\text{s.t.} \quad e_i = \frac{1}{c} \beta_i g_i, \quad e^{-i} = \frac{1}{c} \beta_{-i} g^{-i}, \quad (39)$$

$$0 \leq \alpha_i + \beta_i g^T e - \frac{c}{2} e_i^2, \quad (40)$$

$$0 \leq \alpha_{-i} + \beta_{-i} g^T e - \frac{c}{2} (e^{-i})^T e^{-i}. \quad (41)$$

The solution to this problem is straightforward and leads to optimal explicit bonuses of $\beta_i = f_i/g_i$, $\beta_{-i} = (f^{-i})^T g^{-i}/((g^{-i})^T g^{-i})$ and an expected profit of

$$\bar{\pi} := \frac{f_i^2}{2c} + \frac{\|f_{-i}\|^2}{2c} \cos^2 \theta_{-i}. \quad (42)$$

Thus, the optimal combination of explicit and relational contracts is determined by solving

$$\max_{\substack{\alpha_l, \beta_l, \gamma_l, e_l \\ l=i, -i}} f^T e - \sum_{l=i, -i} (\alpha_l + \beta_l g^T e + \gamma_l f^T e), \quad (43)$$

$$\text{s.t.} \quad e_i = \frac{1}{c} (\gamma_i f_i + \beta_i g_i), \quad (44)$$

$$e^{-i} = \frac{1}{c} (\gamma_{-i} f^{-i} + \beta_{-i} g^{-i}), \quad (45)$$

$$0 \leq \alpha_i + \beta_i g^T e + \gamma_i f^T e - \frac{c}{2} e_i^2, \quad (46)$$

$$0 \leq \alpha_{-i} + \beta_{-i} g^T e + \gamma_{-i} f^T e - \frac{c}{2} (e^{-i})^T e^{-i}, \quad (47)$$

$$\phi(\gamma_i + \gamma_{-i}) \leq f^T e - \sum_{l=i, -i} (\alpha_l + \beta_l g^T e + \gamma_l f^T e) - \bar{\pi}. \quad (48)$$

Note that (48) also implies that the principal will not break the relational contract with only one of the two agents. It is easy to verify that the agents' participation constraints (46) and (47) bind at the optimal solution. Thus, by substituting e_i and e^{-i} and defining

$$\begin{aligned} \pi(\beta_i, \gamma_i, \beta_{-i}, \gamma_{-i}) &:= \frac{1}{c} [f_i(\beta_i g_i + \gamma_i f_i) + (f^{-i})^T (\beta_{-i} g^{-i} + \gamma_{-i} f^{-i})] - \\ &\quad \frac{1}{2c} [(\beta_i g_i + \gamma_i f_i)^2 + (\beta_{-i} g^{-i} + \gamma_{-i} f^{-i})^T (\beta_{-i} g^{-i} + \gamma_{-i} f^{-i})], \end{aligned} \quad (49)$$

the problem can be simplified to

$$\max_{\substack{\beta_i, \gamma_i \\ \beta_{-i}, \gamma_{-i}}} \pi(\beta_i, \gamma_i, \beta_{-i}, \gamma_{-i}) \quad (50)$$

$$\text{s.t. } \phi(\gamma_i + \gamma_{-i}) \leq \pi(\beta_i, \gamma_i, \beta_{-i}, \gamma_{-i}) - \frac{f_i^2}{2c} - \frac{\|f^{-i}\|^2}{2c} \cos^2 \theta_{-i}. \quad (51)$$

From the first-order condition for β_i ,

$$\beta_i(\gamma_i) = (1 - \gamma_i) \frac{f_i}{g_i} \text{ for } 0 \leq \gamma_i \leq 1. \quad (52)$$

It follows from (44) that $e_i = e_i^{FB}$. After substituting β_i , the principal's problem becomes

$$\max_{\substack{\gamma_i \\ \beta_{-i}, \gamma_{-i}}} \frac{f_i^2}{2c} + \pi(0, 0, \beta_{-i}, \gamma_{-i}) \quad (53)$$

$$\text{s.t. } \phi(\gamma_i + \gamma_{-i}) \leq \pi(0, 0, \beta_{-i}, \gamma_{-i}) - \frac{\|f^{-i}\|^2}{2c} \cos^2 \theta_{-i}. \quad (54)$$

Thus, the principal cannot do better than setting $\gamma_i = 0$. The remaining optimization problem corresponds to the one considered in section 3 so that $\pi^S(\gamma_{-i}^*)$ follows from (20).

It remains to show that $\gamma_{-i}^* \leq \gamma^*$. By (19), this will be the case if

$$\|f^{-i}\|^2(1 - \cos^2 \theta_{-i}) \leq \|f\|^2(1 - \cos^2 \theta). \quad (55)$$

For parsimony, define $x := \cos^2 \theta$ and $x_{-i} := \cos^2 \theta_{-i}$. Then, (55) is equivalent to

$$\begin{aligned} \|f\|^2 x &\leq f_i^2 + \|f^{-i}\|^2 x_{-i} \\ \Leftrightarrow \frac{(f_i g_i + f_j g_j + f_k g_k)^2}{g_i^2 + g_j^2 + g_k^2} &\leq f_i^2 + \frac{(f_j g_j + f_k g_k)^2}{g_j^2 + g_k^2}. \end{aligned} \quad (56)$$

By defining $A := f_j g_j + f_k g_k$ and $B := g_j^2 + g_k^2$ we receive

$$\frac{(f_i g_i + A)^2}{g_i^2 + B} \leq f_i^2 + \frac{A^2}{B}. \quad (57)$$

This inequality can be transformed to

$$0 \leq g_i^2 - 2f_i g_i \frac{B}{A} + f_i^2 \frac{B^2}{A^2} = \left(g_i - f_i \frac{B}{A} \right)^2. \quad (58)$$

Thus, $\gamma_{-i}^* \leq \gamma^* < 1$, i.e., $(e^{FB})^{-i}$ is implemented if and only if $\cos \theta_{-i} = 1$ (by setting $\gamma_{-i}^* = 0$ and $\beta_{-i}^* = \frac{\|f^{-i}\|}{\|g^{-i}\|}$).²⁸ \square

Proof of proposition 4. (i) By proposition 3, from $\gamma^* = 0$ it follows that $\gamma_{-i}^* = 0$.

Thus, task splitting leads to a weakly higher expected profit iff

$$\frac{\|f\|^2}{2c}x \leq \frac{f_i^2}{2c} + \frac{\|f^{-i}\|^2}{2c}x_{-i} \quad (59)$$

which holds by (56). \square

(ii) By proposition 3, $\gamma^* \geq \gamma_{-i}^*$. By assumption, $\gamma^* < 1$. Thus, task splitting leads to a lower expected profit than no task splitting iff

$$\begin{aligned} \frac{\|f\|^2 - \|f^{-i}\|^2}{2c} + \frac{\|f^{-i}\|^2}{2c}x_{-i} + 2\phi \left(1 - \frac{c\phi}{\|f^{-i}\|^2(1 - x_{-i})}\right) \leq \\ \frac{\|f\|^2}{2c}x + 2\phi \left(1 - \frac{c\phi}{\|f\|^2(1 - x)}\right). \end{aligned} \quad (60)$$

Because $\gamma_{-i}^* < 1$, we have $\|f^{-i}\|^2(1 - x_{-i}) < 2c\phi$ and, therefore, the left-hand side of (60) strictly increases in $\|f^{-i}\|^2(1 - x_{-i})$. Furthermore, (60) binds iff $\|f^{-i}\|^2(1 - x_{-i}) = \|f\|^2(1 - x)$. Thus, by (55), (60) holds. \square

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²⁸From $\gamma_{-i}^* < 1$ it follows that $\gamma_i^* = 0$ is the uniquely optimal implicit bonus for agent 1.

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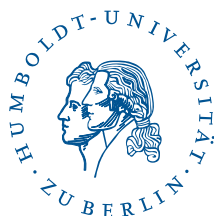
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